



A Data Processing Approach to High Precision , High Return Rate kHz SLR Stations

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Statement of the Problem



- kHz Single photon sensitive SLR stations are typically operated at low return rates ($\sim 10\%$) to minimize range biases due to “first photon effects”.
- Operation at low return rates partially negates one of the advantages of kHz systems, i.e. the ability to form accurate normal points more quickly, thereby reducing overall data volume by an order of magnitude or more and limiting the ability to move rapidly between satellites.
- At the 2017 ILRS Technical Workshop in Riga, the author proposed that the use of centroid detection circuits, instead of legacy threshold detection systems, would allow the rapid production of bias-free normal points independent of signal return rates.
- The present paper investigates an alternative software approach for removing rate-dependent range bias which is applicable to commonly used threshold detection systems.
- Signal detection is modelled as a Two State Markov Process, and the return rate within a given Normal Point (NP) is used, via Poisson statistics, to estimate the number of 1, 2, 3..etc photoelectron events contributing to the NP and to correct for the range bias.
- The single photon Probability Distribution Function (PDF) for the instrument ranging to a target (calibration or satellite) can be obtained theoretically or experimentally and used to correct for biases at high return rates.



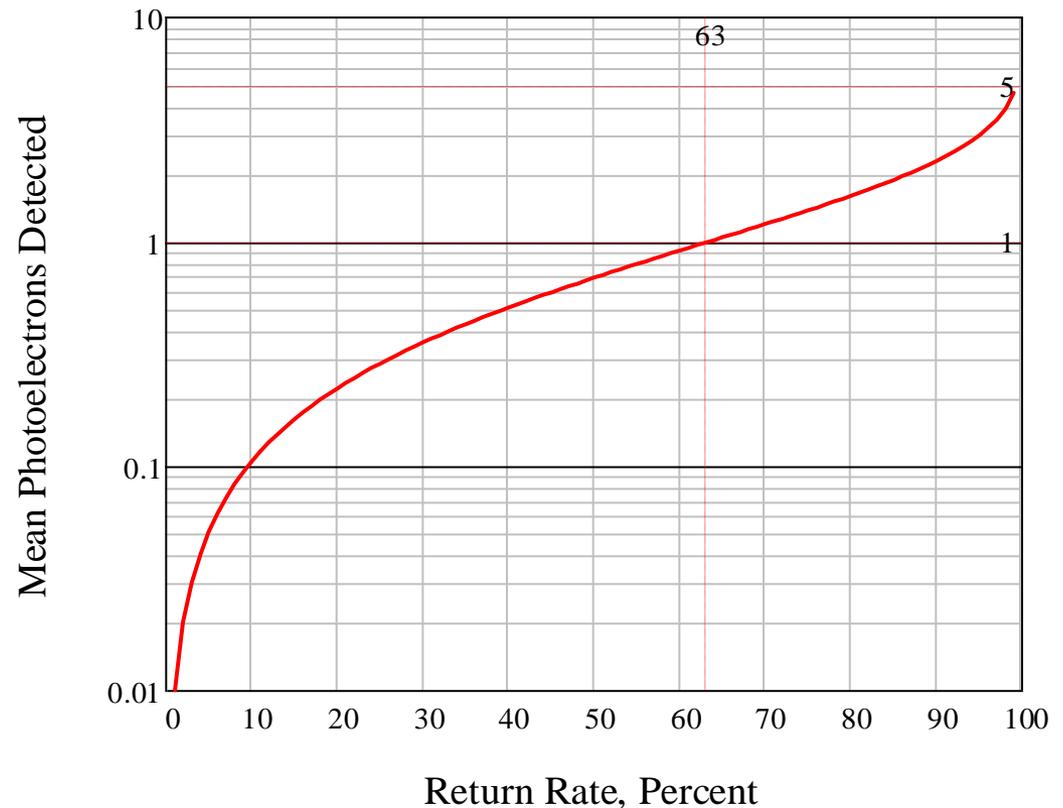
For an SLR receiver having a single photon detection threshold, the probability of detecting the satellite signal is given by Poisson statistics as:

$$P_D = 1 - \exp(-\eta)$$

where η is the mean number of photoelectrons detected per pulse. Solving for η yields

$$\eta = \ln\left(\frac{1}{1 - P_D}\right)$$

If $\eta = 1$, PD = 0.63 and the return rate is 63%. For $\eta \geq 5$, PD ~ 1 and the return rate is $\sim 100\%$.





From Poisson statistics, the probability that a given return within the NP consists of n photoelectrons when the mean number is η is given by

$$P(n, \eta) = e^{-\eta} \frac{\eta^n}{n!}$$

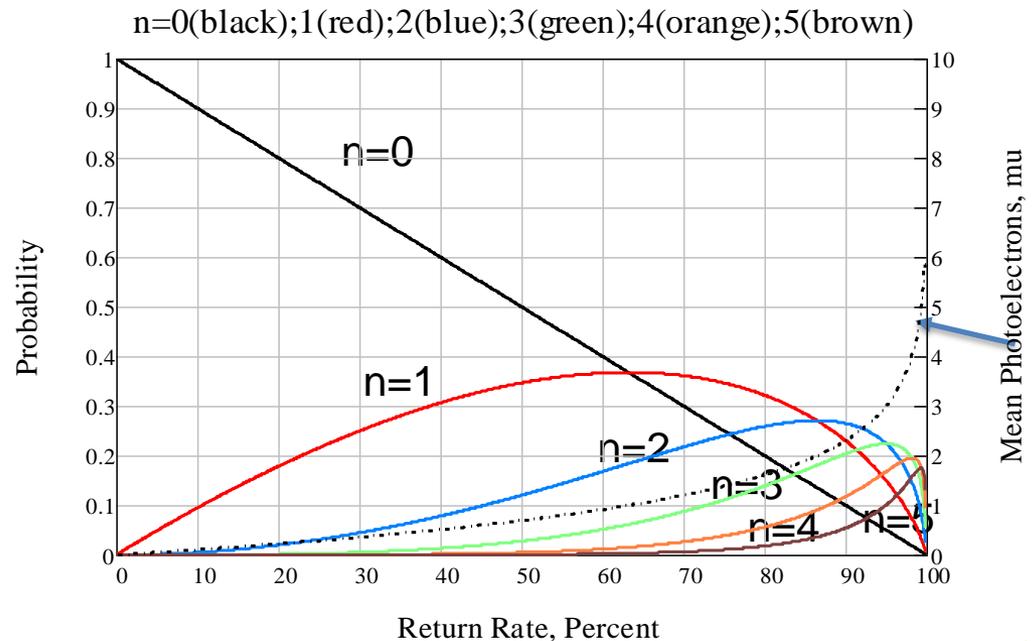
and the total probability of detecting the signal (i.e. the return rate) is equal to

$$P_D(\eta) = \sum_{n=1}^{\infty} P(n, \eta)$$
$$= e^{-\eta} \sum_{n=1}^{\infty} \frac{\eta^n}{n!} = 1 - e^{-\eta}$$

where the mean signal strength (η) and Return Rate (RR) are equal to

$$\eta = \ln\left(\frac{1}{1 - P_D}\right)$$

$$RR = P_D(100\%)$$





SGSLR Link Equation



$$n_s = \frac{E_t}{h\nu} \eta_t \frac{2}{\pi(\theta_d R)^2} \exp \left[\underset{\text{Pointing Bias}}{-2 \left(\frac{\Delta\theta_p}{\theta_d} \right)^2} \right] \left[\underset{\text{Pointing Jitter}}{\frac{1}{1 + \left(\frac{\Delta\theta_j}{\theta_d} \right)^2}} \right] \left(\frac{\sigma A_r}{4\pi R^2} \right) \eta_r \eta_c T_a^2 T_c^2$$

To compute the expected satellite return rates, we use a comprehensive link equation which includes:

- Relevant station and satellite parameters
- Telescope Pointing Bias and Jitter
- Atmospheric Visibility
- Mean Cirrus Cloud Transmission
- Atmospheric turbulence effects and target speckle

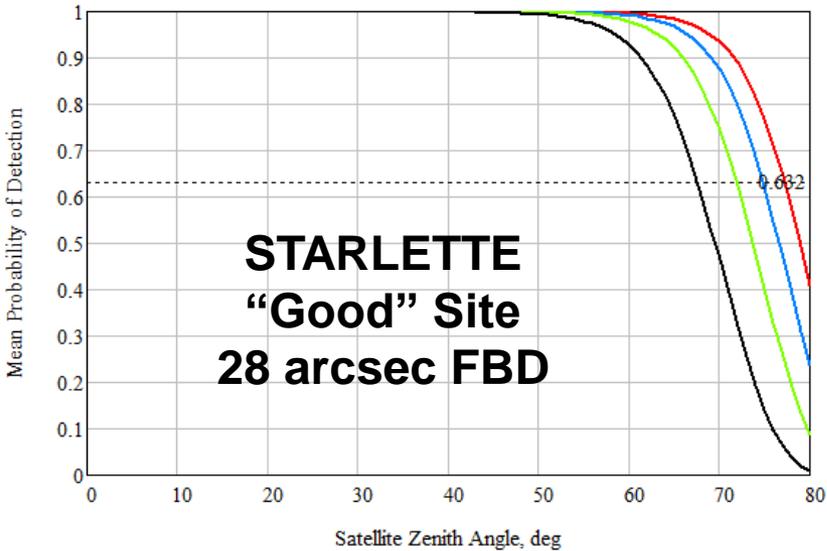
VARIABLE	SYMBOL	SGSLR VALUE
Laser Pulse Energy	E_t	1.5 mJ (60% of Max)
Laser Repetition Rate	f_L	2 kHz
Transmit Optics Efficiency	η_t	0.766
Receive Optics Efficiency	η_r	0.542
Detector Counting Efficiency	η_c	0.28
Spectral Filter Efficiency	η_f	0.7
Effective Receive Aperture	A_r	0.187 m ²
Tracking Pointing Bias	$\Delta\theta_p$	2 arcsec (Sigma Range Receiver)
Telescope RMS Pointing Jitter	$\Delta\theta_j$	2 arcsec
Full Transmitter Divergence	$2\theta_d$	28 arcsec (Starlette, LAGEOS) 14 arcsec (GNSS)
Coherence Length	ρ_0	20 cm (Excellent Site) 10 cm (Good Site) 2.5 cm (Poor Site)



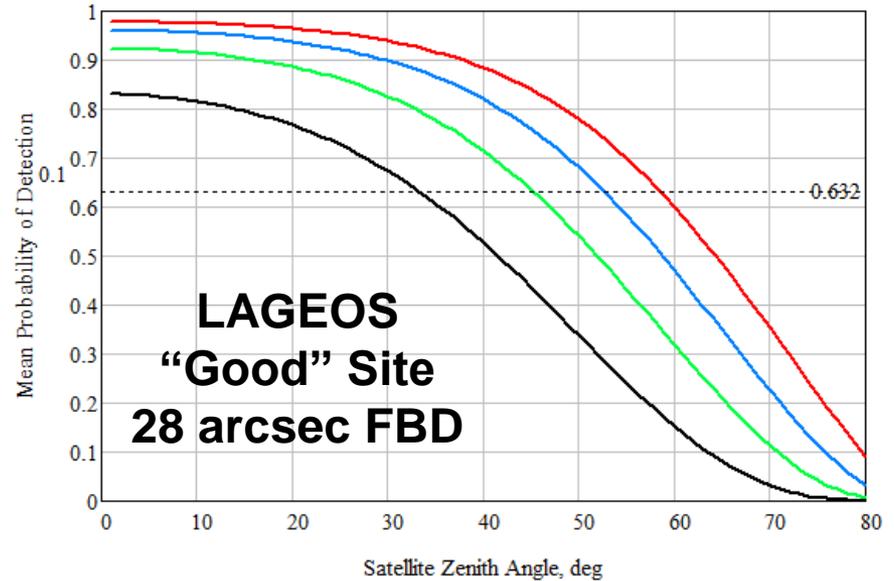
SGSLR Link Analysis



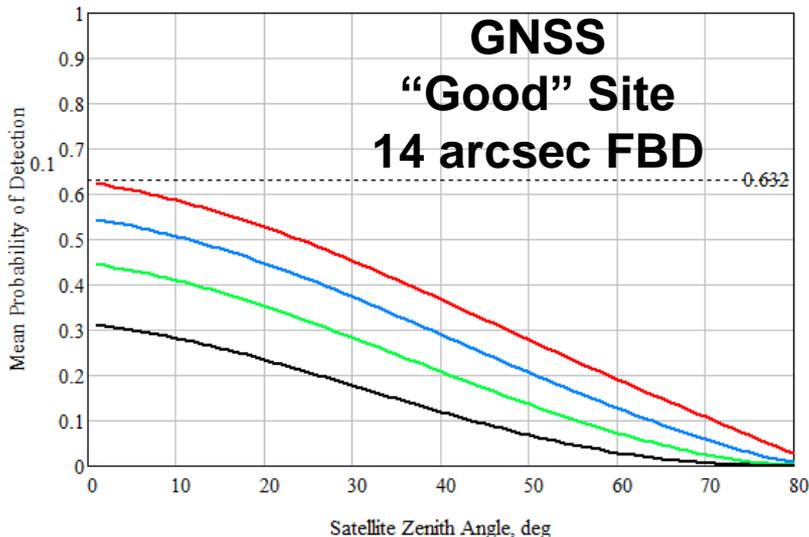
Visibility: Red = Ext. Clear; Blue =Std Clear; Green = Clear, Black= Light Fog



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Visibility: Red = Ext. Clear; Blue =Std Clear; Green = Clear, Black= Light Fog



FBD = Full Beam Divergence

Conclusion: For low satellite zenith angles ($<40^\circ$) and reasonable atmospheric visibility ($V > 15$ km), the return rate can greatly exceed the nominal "bias-free" 10% rate for all satellites leading to much shorter mm normal point integration times and much greater data volumes.



The PDF for the photon time of arrival at the receiver is obtained by convolving the PDFs of the laser (L), the target (T), and the receiver (R), i.e.

$$\lambda(t) = L * T * R$$

Thus, the photoelectrons arriving at the receiver have a PDF given by

$$\lambda(t) = \int_{-\infty}^t dt' R(t-t') \int_{-\infty}^{t'} dt'' L(t'') T(t'-t'')$$

which, for a single retro calibration target having a delta function response, reduces to

$$\lambda_c(t) = \int_{-\infty}^t dt' R(t-t') \int_{-\infty}^{t'} dt'' L(t'') \delta(t'+\tau_c-t'') = \int_{-\infty}^t dt' R(t-t') L(t'+\tau_c)$$

where τ_c is the roundtrip flight time to the target. The instrument PDF due to the laser and receiver, $\lambda_c(t)$, can be measured at the output of the detector with a high speed oscilloscope or, for ultrashort pulses, a sampling scope, but we will also provide an experimental alternative for determining $\lambda_c(t)$.

For a spherical geodetic satellite, the impulse response can be described by*

$$I(\tau, \varepsilon, n_{cc}, \theta_{max}) = \sigma_{cc} \frac{N}{2} \sin \theta(\tau, \varepsilon, n_{cc}) \left[1 - \frac{\theta(\tau, \varepsilon, n_{cc})}{\theta_{max}} \right]^2$$

where σ_{cc} is the optical cross-section of a single cube corner, N is the number of cube corners uniformly distributed over the spherical surface, n_{cc} is the refractive index of the cube corner, $\varepsilon = n_{cc}L_{cc}/R_s$ is the ratio of the optical length (face to vertex) of an individual cube ($n_{cc}L$) to the satellite radius R_s , $\tau = ct/2R_s$ is a normalized time expressed in units of the roundtrip transit time of the laser pulse from the surface of the satellite to the satellite center of mass and back, θ_{max} is the maximum acceptance angle of the retroreflector from normal incidence. The quantity $\theta(\tau, \varepsilon, n_{cc})$ is obtained by solving the equation

$$\cos \theta(\tau, \varepsilon, n_{cc}) = \frac{1 - \tau}{1 - \varepsilon \sqrt{1 - \frac{1}{n_{cc}^2} + \left(\frac{\cos \theta(\tau, \varepsilon, n_{cc})}{n_{cc}} \right)^2}}$$

as a function of τ . The target PDF $T(t)$, used in computing of $\lambda(t)$, is $I(\tau, \varepsilon, n_{cc})$ whose integral is normalized to 1.

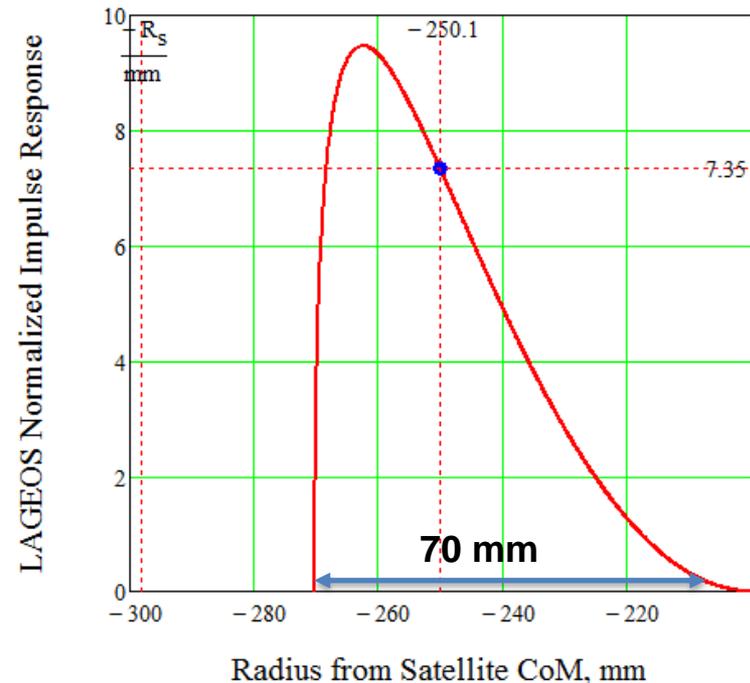
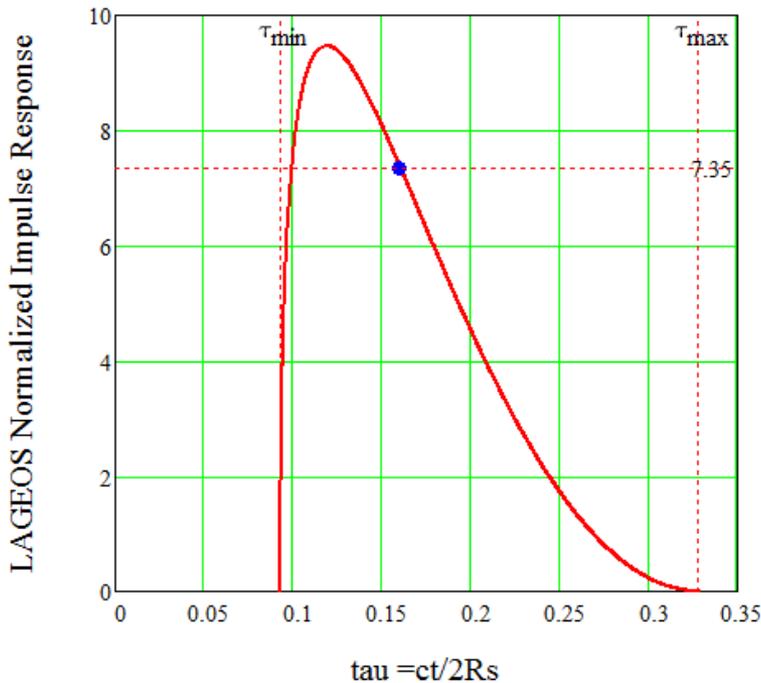
* See J. Degnan, "Millimeter Accuracy Satellite Laser Ranging: A Review", Contributions of Space Geodesy to Geodynamics: Technology, D. E. Smith and D. L. Turcotte (Eds.), AGU Geodynamics Series, Volume 25, pp. 133-162



LAGEOS Impulse Response



The graphs below present the normalized impulse response for LAGEOS, i.e. the target PDF $T(t)$ used in the generation of $\lambda(t)$, expressed as a function of $\tau = ct/2R_s$ on the left and distance of the return from the satellite Center of Mass (CoM) on the right. The centroid of the PDF is indicated by the blue dots, i.e. $\tau_c = 0.16$ corresponding to a satellite radius from CoM of ~ 250 mm in good agreement with LAGEOS lab measurements. The total width (zero to zero) of the LAGEOS impulse response is about 70 mm or 468 psec.



Threshold detection can be treated as a Two State Markov Process with the initial state being “no detection” and the final state being “detection” (if $n > 0$). The time of detection PDF depends on the detection threshold, T , the number of photoelectrons detected, n , and the n -photon temporal PDF distribution $\lambda(t)$ given by

$$P_n(t) = \frac{1}{1 - e^{-n}} a(n, T, t) \exp \left[- \int_{t_0}^{t_f} dt' a(n, T, t') \right]$$

where

$$a(n, T, t) = n\lambda(t) \frac{[n\lambda(t)]^{T-1}}{(T-1)!} \left\{ \sum_{k=0}^{T-1} \frac{[n\lambda(t)]^k}{k!} \right\}^{-1}$$

For a single photon detection threshold ($T = 1$) as in kHz SLR systems,

$$a(n, 1, t) = n\lambda(t) \quad \text{and} \quad P_n(t) = \frac{\mu_n(t)}{1 - e^{-n}} = \frac{1}{1 - e^{-n}} n\lambda(t) \exp \left[-n \int_{t_0}^t dt' \lambda(t') \right]$$

where $\lambda(t)$ has a non-zero value only in the time interval $t_0 < t < t_f$ and the integral of $\lambda(t)$ over that interval is equal to 1.

Note: once $\lambda(t)$ is known, the functional form of $\mu_n(t)$ is determined for all values of n .

For a normal point generated with multiple values of n and having a mean signal strength η , the bias in the photon time of detection is

$$\Delta t(\eta) = \langle t(\eta) \rangle - \langle t_0 \rangle = \sum_{n=1}^{\infty} P(n, \eta) \langle t_n \rangle - \langle t_0 \rangle = e^{-\eta} \sum_{n=1}^{\infty} \frac{\eta^n}{n!} \langle t_n \rangle - \langle t_0 \rangle$$

where the centroid of the PDF for n detected photoelectrons is given by

$$\langle t_n \rangle \equiv \int_{t_0}^{t_f} dt t P_n(t) = \frac{1}{1 - e^{-n}} \int_{t_0}^{t_f} dt t \mu_n(t) = \frac{n}{1 - e^{-n}} \int_{t_0}^{t_f} dt t \lambda(t) \exp \left[-n \int_{t_0}^t dt' \lambda(t') \right]$$

which, in the limit as n goes to zero, reduces to the unbiased photon arrival time

$$\langle t_0 \rangle = \int_{t_0}^{t_f} dt t \lambda(t)$$



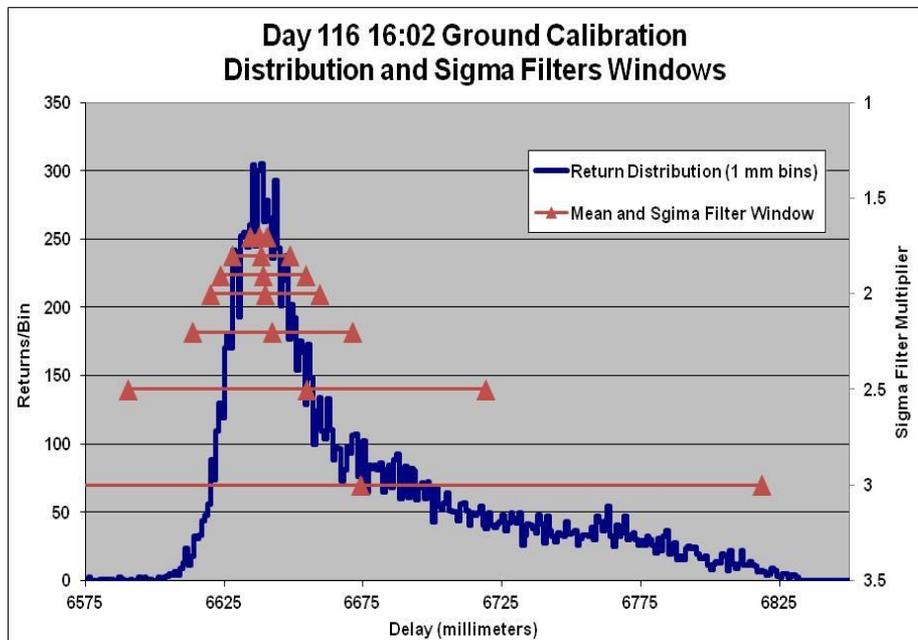
Determining $\lambda(t)$ Experimentally



Another way to measure the function $\lambda(t)$ from range data to the calibration target (or even a satellite) is to utilize a low return rate (<10%) such that one is always seeing single photon returns. In this instance, the PDF of the measured ranges should obey the functional form

$$P_1(t) = \frac{\mu_1(t)}{1 - e^{-1}} = \frac{1}{1 - e^{-1}} \lambda(t) \exp \left[- \int_{t_0}^t dt' \lambda(t') \right] \quad \text{where } t_0 \leq t \leq t_f$$

and t_0 and t_f are defined as the end points of the $n=1$ detection PDF where $\lambda(t)=0$. The following graph shows the unsmoothed single photon PDF, $P_1(t)$, for NASA's prototype NGSLR station ranging to the calibration target.



The profile $P_1(t)$ can be smoothed (for example) by: (1) computing the Fourier Transform, (2) applying a bandpass filter to eliminate high frequency noise, and then (3) computing the inverse Fourier transform to provide the function $\mu_1(t)$ in tabulated or functional form. This can then be used to compute $\lambda(t)$ and the PDFs, $P_n(t)$, for a small range of n values and correct for biases in all future measurements to the same target!

We begin by computing $\mu_1(t)$ from the observed single photon PDF $P_1(t)$

$$\mu_1(t) = (1 - e^{-1}) P_1(t) \equiv \lambda(t) \exp\left[-\int_{t_0}^t \lambda(t') dt'\right] = -\frac{d}{dt} \exp\left[-\int_{t_0}^t \lambda(t') dt'\right]$$

Integrating both sides of the equation with respect to t yields

$$\exp\left[-\int_{t_0}^t \lambda(t') dt'\right] = 1 - \int_{t_0}^t \mu_1(t') dt'$$

Computing the logarithm of both sides gives

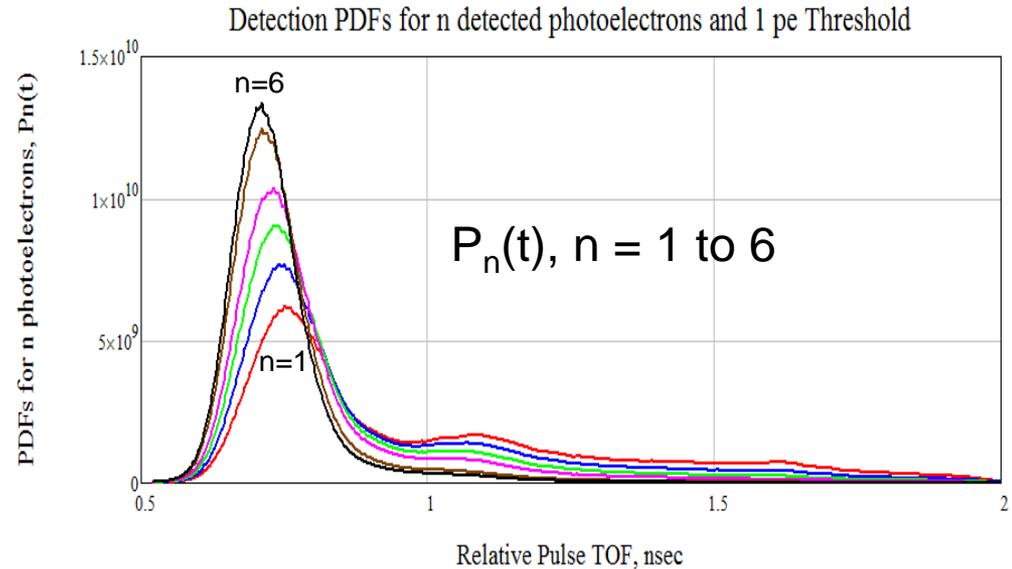
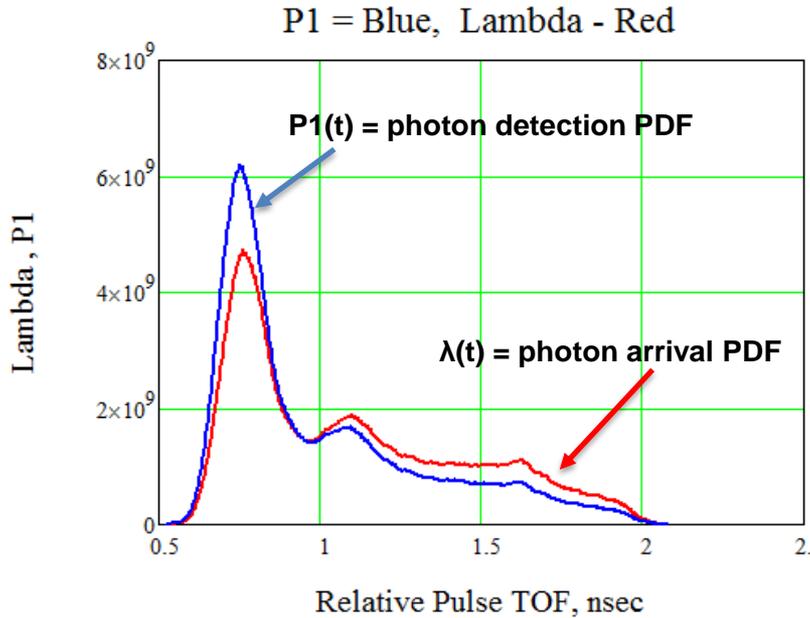
$$\int_{t_0}^t \lambda(t') dt' = \ln\left(\frac{1}{1 - \int_{t_0}^t \mu_1(t') dt'}\right)$$

and differentiating both sides with respect to t yields our final result

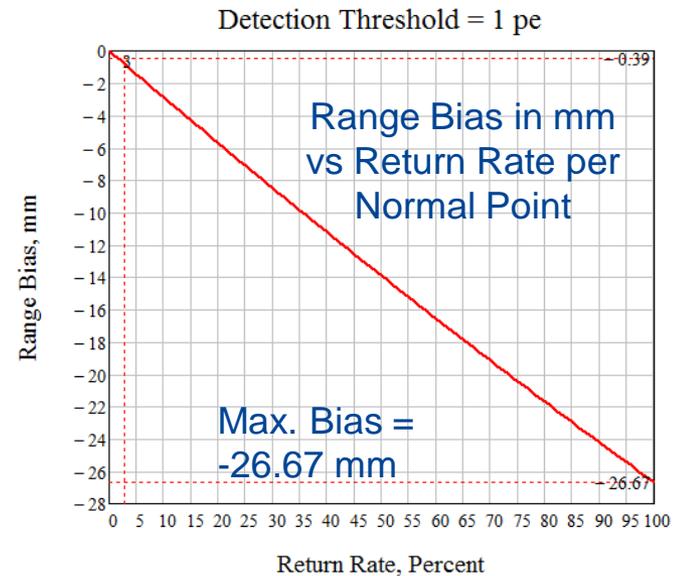
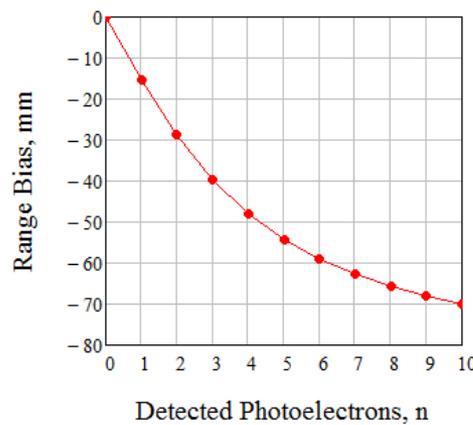
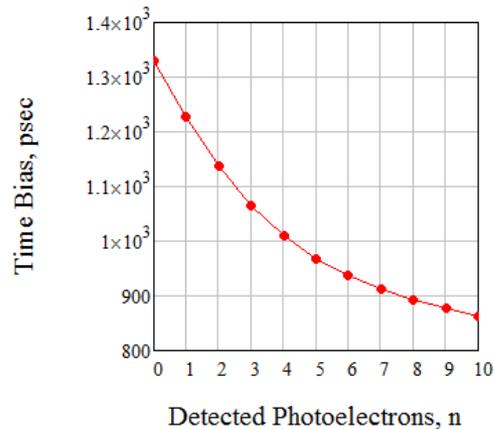
$$\lambda(t) = \frac{\mu_1(t)}{1 - \int_{t_0}^t \mu_1(t') dt'}$$



NGSLR to Calibration Target

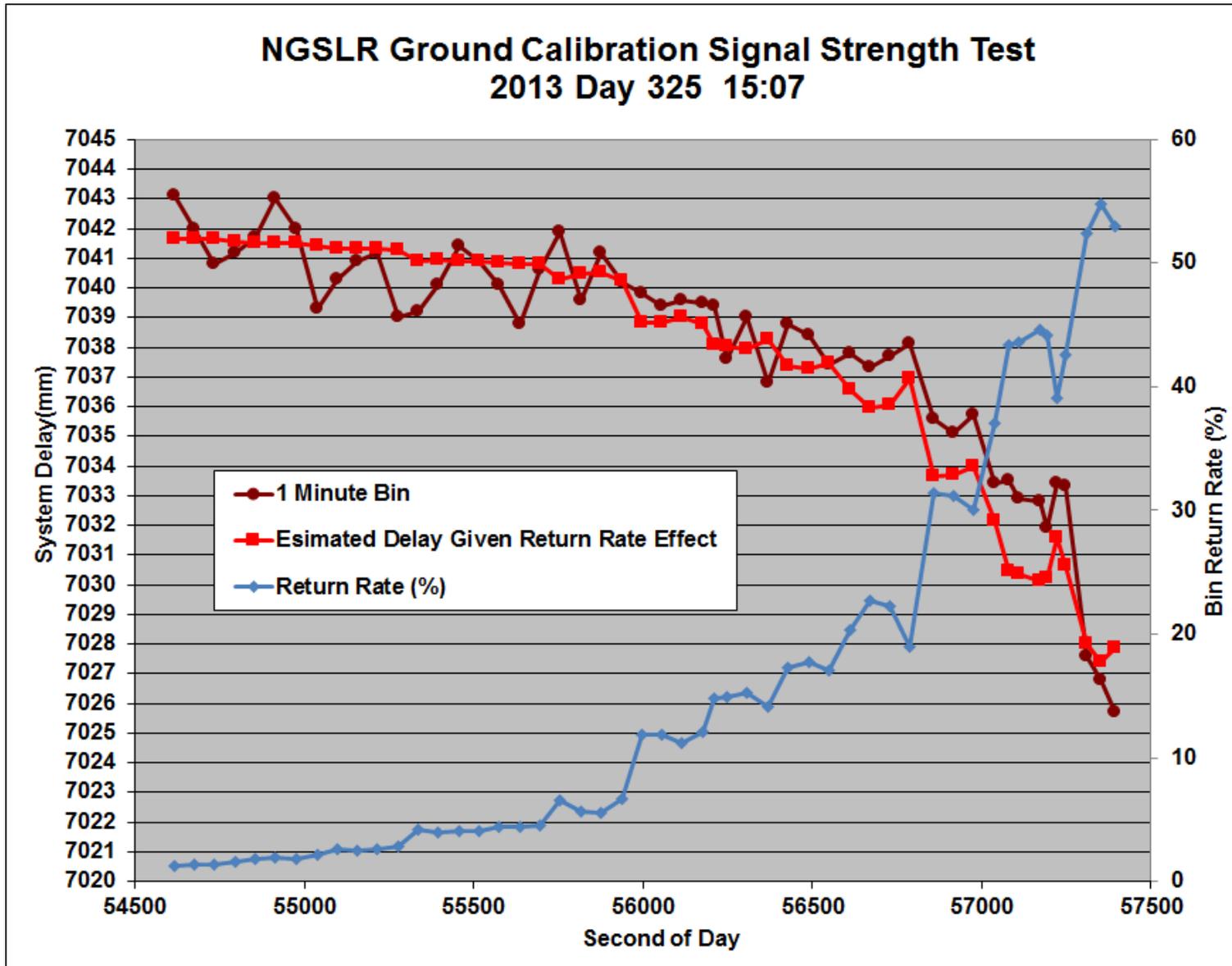


Time and Range Bias vs Number of Detected Photoelectrons (n = 0 to 10)



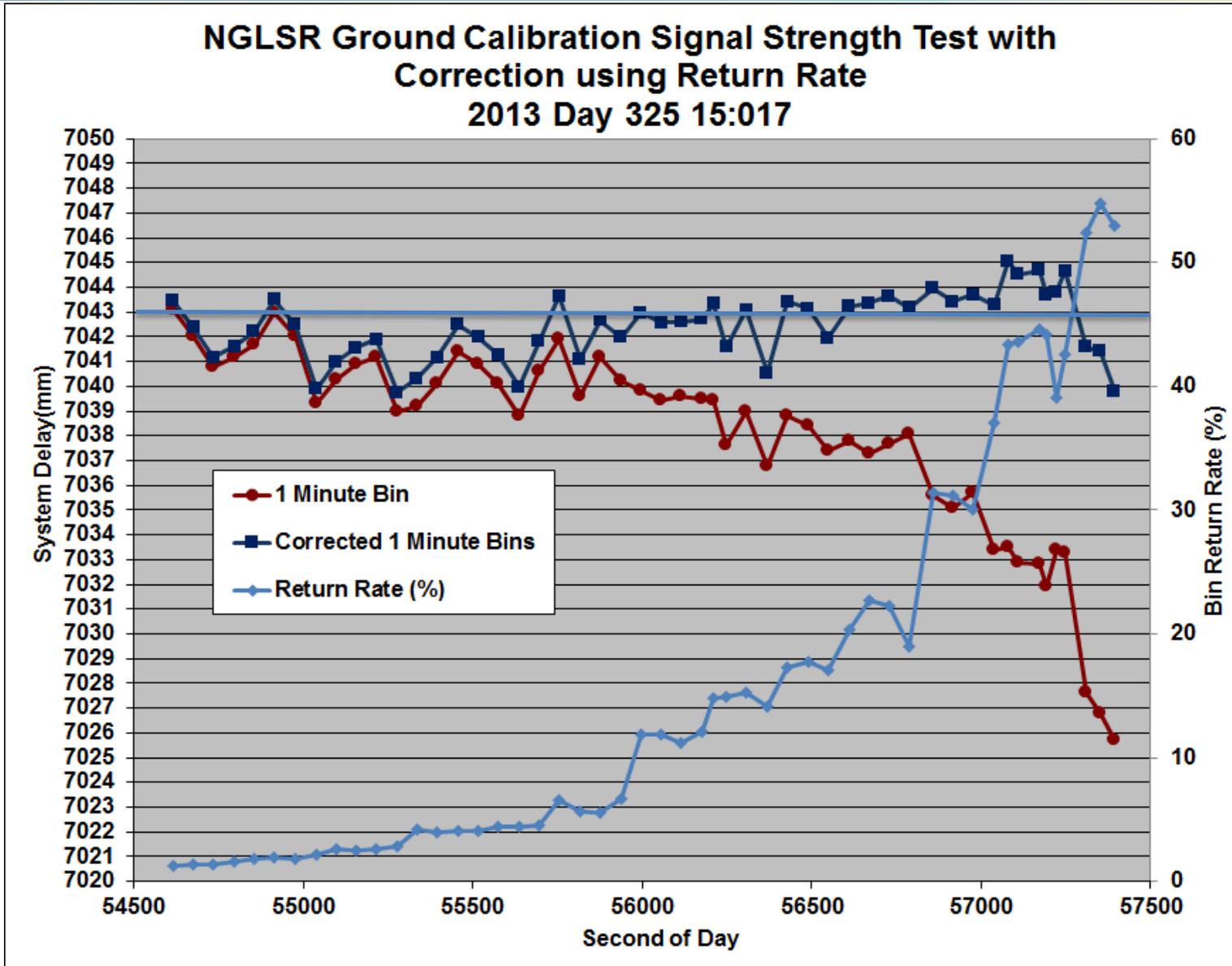


NGSLR Calibration Test Theory vs Experiment





NGSLR Calibration Target Test Theory vs Experiment

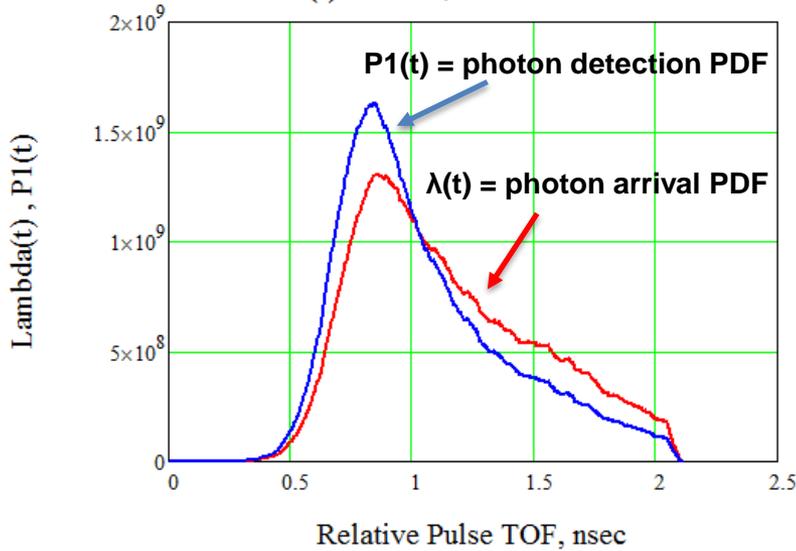




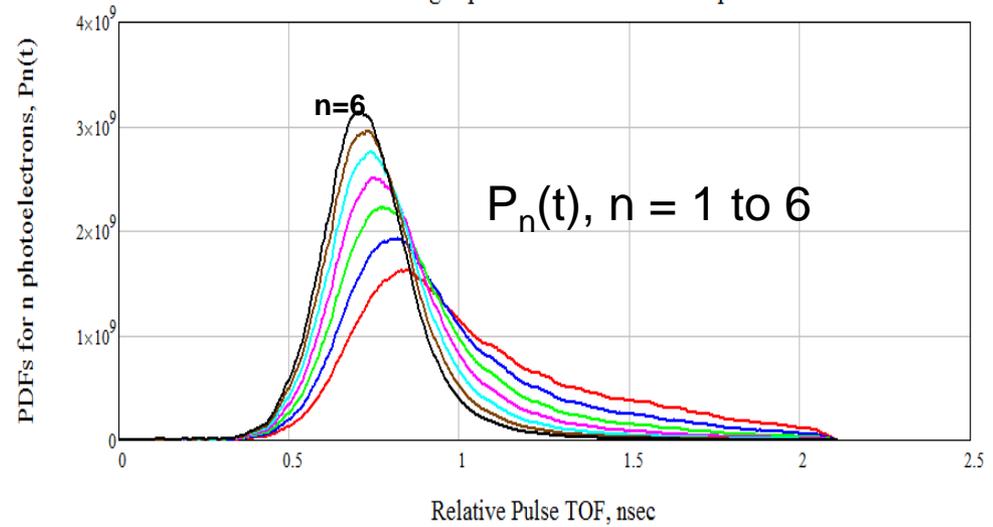
NGSLR to LAGEOS



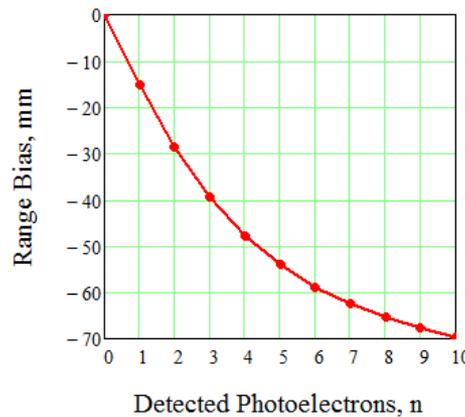
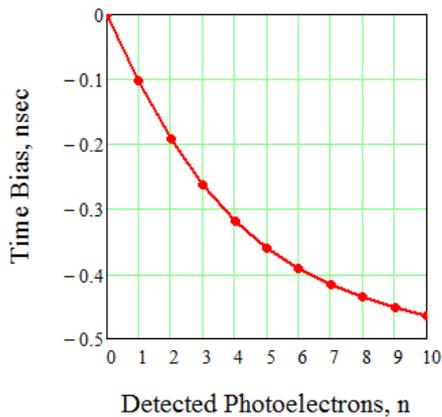
$P_1(t) = \text{Blue, } \lambda(t) = \text{Red}$



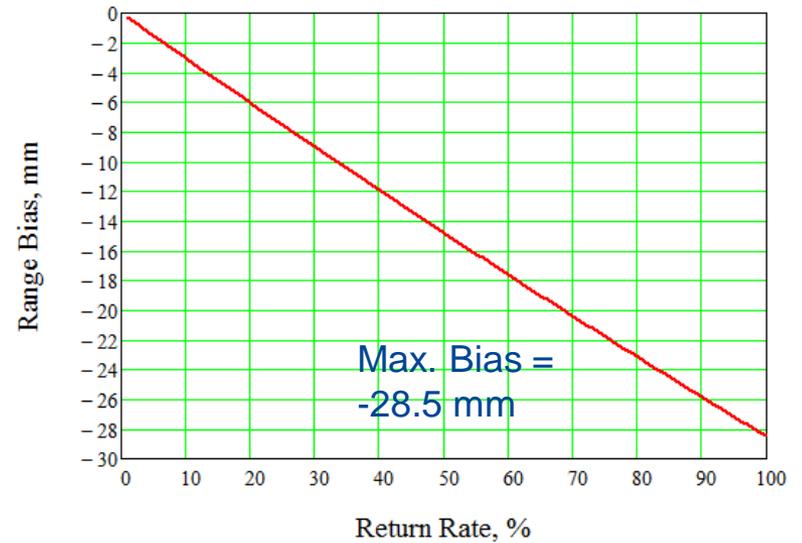
PDFs for detecting n photoelectrons with a 1 pe threshold



Time and Range Bias vs Number of Detected Photoelectrons ($n = 0$ to 10)



NGSLR/LAGEOS Range Bias vs Return Rate





Summary

- **We have proposed both a theoretical and an experimental method for correcting the range bias in a normal point for an arbitrary return rate.**
- **This method not only provides a potentially bias-free range measurement but also removes the restriction to use only low return rates** thereby greatly
 - Reducing the integration time for normal point generation and reducing the length of the orbital path which defines that normal point.
 - Enhancing satellite data volumetric output in kHz SLR systems
 - Speeding up the interleaving of satellites.
- The **theoretical method** determines the function $\lambda(t)$ by convolving the known PDFs for the laser, target, and receiver and then uses the result to compute the various PDFs associated with higher values of n and their corresponding time or range centroids, t_n .
- The **experimental method** uses low return rate measurements (<10%) to a particular target (calibration or satellite) to determine the single pe PDF $P_1(t)$ for that target and again uses that result to compute the PDFs and centroids for higher values of n . High frequency noise in the experimental data can be removed by a smoothing method, e.g. computing the Fourier transform, applying a bandwidth filter, and performing an inverse Fourier Transform.
- **The approach assumes that the target response is largely independent of viewing angle, as with** uniformly populated spherical geodetic satellites (LAGEOS, Starlette, etc.) or remote sensing or GNSS satellites where legacy flat panel arrays are replaced by segments of uniformly populated spheres. (See J. Degnan, "Reducing the Satellite Contribution to Range Error", 20th International Workshop on Laser Ranging, Potsdam, Germany, October 2016.)
- Our results to date using NGSLR data suggest that the range bias is expected to vary linearly from 0 at very low return rates to a maximum on the order of -27 mm at very high return rates near 100%.
Thus, two bias measurements at a very low and very high rate can define the bias at all rates.

ACKNOWLEDGEMENT. The author wishes to thank Christopher B. Clarke of KBRwyle for smoothing the low data rate NGSLR returns from the calibration target and the LAGEOS satellite and plotting the final comparisons of the high signal data biases with the theoretical predictions.¹⁸